

CHAPTER TWO

DIMENSIONAL ANALYSIS, SIMILITUDE & HYDRAULIC MODELS

2.1. INTRODUCTION

Hydraulics is an experimental science & is also a complex subject. Most phenomena involving the movement of water are known to be dependent on many variables- geometric characteristics, fluid properties and flow characteristics. Much of raw experimental data were incomplete, ambiguous, & incorrectly taken.

It is usually impossible to determine all the essential facts for a given fluid flow by pure theory, & hence, dependence must often be placed up on experimental investigations.

The number of tests to be made can be greatly reduced by the systematic use of Dimensional Analysis and the laws of similitude or similarity. Dimensional Analysis is a mathematical technique, which makes use of the study of dimension as an aid to the solution of several engineering problems

Dimensionless grouping reduces the number of variables that have to be processed. Dimension analysis can be used to obtain functional relationship among the variables in terms of non-dimensional parameter. It helps in obtaining a systematic form of the variables involved in the problem. However, dimensional analysis doesn't give complete relationship; it gives only a general relationship.

Some of the uses of dimensional analysis are narrated as follows: -

- i. Testing the dimensional homogeneity of any equation of fluid motion
- ii Deriving equations expressed in terms of non-dimensional parameters to show the significance of each parameter.
- iii planning model tests & presenting experimental results in a systematic manner in terms of non-dimensional parameters; thus making it possible to analyse the complex fluid flow phenomena.

Hydraulic structures or machines can be designed using:

- (i) Pure theory,
- (ii) Empirical methods,
- (iii) Semi-empirical methods which are mathematical formulations based on theoretical concepts supported by suitably designed experiments,
- (iv) Physical models,
- (v) Mathematical models.

i) **The purely theoretical** approach in hydraulic engineering is limited to a few cases of laminar flow, for example the Hagen Poiseuille equation for the hydraulic gradient in the laminar flow of an incompressible fluid in a circular pipeline.

ii) **Empirical methods** are based on correlations between observed variables affecting a particular physical system. Such relationships should only be used under similar circumstances to those under which the data were collected. Due to the inability to express the physical interaction of the parameters involved in mathematical terms some such methods are still in use. One well-known example is in the relationship between wave heights, wind speed and duration for the forecasting of ocean wave characteristics.

iii) A good example of a **semi-empirical** relationship is the Colebrook white equation for the friction factors in turbulent flow in pipes (We will see it in chapter 4). This was obtained from theoretical concepts and experiments designed on the basis of dimensional analysis; it is universally applicable to all Newtonian fluids.

iv) Dimensional analysis also forms the basis for the design and operation of **physical scale** models, which are used to predict the behavior of their full –sized counterparts called ‘prototypes’. Such models, which are generally geometrically similar to the prototype, are used in the design of aircraft, ships, submarines, pumps, turbines, harbors, breakwaters, river and estuary engineering works, spillways, etc.

V) The **mathematical modeling** techniques have progressed rapidly due to the advance of high-speed digital computers, enabling the equations of motion coupled with semi-empirical relationships to be solved for complex flow situations such as pipe network analysis, pressure transients in pipelines, unsteady flows in rivers and estuaries, etc., there are many cases, particularly where localized flow patterns can not be mathematically modeled, when physical models are still needed.

With out the technique of dimensional analysis experimental and computational progress in fluid mechanics would have been considerably retarded.

DIMENSIONAL ANALYSIS

Dimensional analysis is a method, which describes a natural phenomenon by dimensionally correct equation among certain variables which affect the phenomenon. It is a mathematical method, which is of considerable value in problems that occur in fluid mechanics.

In dimensional analysis, from a general understanding of fluid phenomenon, one first predicts the physical parameters that will influence the flow, and then by grouping these parameters in dimension combinations, a better understanding of the flow phenomena is made possible.

Application of dimensional analysis:

- Developing equations –reducing number of variables in an experiment.
- Producing dimensionless parameters – establish the principle of model design.

FUNDAMENTAL DIMENSIONS

In an equation expressing physical relation ship between quantities, absolute numerical and dimension equality must exist. The magnitude of such quantities as mass, length, force, acceleration etc are expressed differently in different units of measurement.

The mass, length, and time are independent of each other and their units of measurement are prescribed by international standards and called fundamental quantities. The units of all other quantities may be determined either from their definitions or from the physical laws describing them.

In place of mass, the force is also sometimes considered as fundamental quantity. The force and the mass are related by the Newton's second law as follows

$$F = m \cdot a$$

$$F = m \cdot a = (M) (LT^{-2})$$

DIMENSIONS OF PHYSICAL QUANTITIES USED IN FLUID MECHANICS

Number	Quantity	Symbol	Dimensions (MLT)
1	Length	L	L
2	Time	T	T
3	Mass	m	M
4	Force	F	$M L T^{-2}$
5	Velocity	V	$L T^{-1}$
6	Acceleration	a	$L T^{-2}$
7	Area	A	L^2
8	Discharge	Q	$L^3 T^{-1}$
9	Pressure	P	$M L^{-1} T^{-2}$
10	Power	P	$M L^2 T^{-3}$
11	Density	ρ	$M L^{-3}$
12	Unit gravity force	γ	$M L^{-2} T^{-2}$
13	Dynamic viscosity	μ	$M L^{-1} T^{-1}$
14	Kinematic viscosity	ν	$L^2 T^{-1}$
15	Surface Tension	σ	$M T^{-2}$
16	Bulk modulus of elasticity	K	$M L^{-1} T^{-2}$
17	Shear stress	$\dot{\epsilon}$	$M L^{-1} T^{-2}$

Dimensional Homogeneity

Any equation describing a physical situation will only be true if both sides have the same dimensions. That is it must be **dimensionally homogenous**. For example the equation which gives for over a rectangular weir (derived earlier in this module) is,

$$Q = \frac{2}{3} B \sqrt{2g} H^{3/2}$$

The SI units of the left hand side are $m^3 s^{-1}$. The units of the right hand side must be the same. Writing the equation with only the SI units gives

$$m^3 s^{-1} = m (m s^{-2})^{1/2} m^{3/2}$$

$$= m^3 s^{-1}$$

i.e. the units are consistent. To be more strict, it is the dimensions which must be consistent (any set of units can be used and simply converted using a constant). Writing the equation again in terms of dimensions,

$$L^3 T^{-1} = L (L T^{-2})^{1/2} L^{3/2}$$

$$= L^3 T^{-1}$$

Notice how the powers of the individual dimensions are equal, (for L they are both 3, for T both -1). This property of dimensional homogeneity can be useful for:

1. Checking units of equations;
2. Converting between two sets of units;

3. Defining dimensionless relationships (see below). k

METHODS OF DIMENSIONAL ANALYSIS

The dimensional analysis is a power full tool in formulating problems of physical phenomenon. It must be solved experimentally. The dimensional analysis accomplishes this by formation of dimension-less groups containing relevant variables. There are several approaches for the dimensional analysis. However, all the methods are absolutely dependent on the correct identification of all the factors, which govern the physical event being analyzed.

The application of dimensional analysis to any practical problem is based on the assumption that certain variable, which affect the phenomenon are independent variables, and all variables other than these and the dependent variable, are irrelevant and have no bearing on the phenomenon.

Steps in dimensional analysis

- ☺ To decide which variables enter the problem
- ☺ Formation of dimensionless groups of the variables
- ☺ Requires thorough understanding
- ☺ Judgment and experience

1. RAYLEIGH'S METHOD

It gives a special form of relationship among the dimensionless group, and has an inherent draw back that it doesn't provide any information regarding the number of dimensionless groups to be obtained as a result of dimensional analysis.

Procedure:

- i. The functional r/s can be written as:

$$Y = f(X_1, X_2, X_3, \dots, X_n)$$
- ii. Any function can be expressed as a series of terms –each being made up of product of variables brought to exponent.

$$Y = K[X_1^a, X_2^b, X_3^c, \dots, X_n^z]$$
- iii. The exponents a, b, c... can be determined by dimensional homogeneity.
- iv. Since there are three fundamental dimensions, only three simultaneous equations can be formed.
- v. The non-dimensional parameters are obtained by grouping variables with like exponent.

Example

A scale model test have been carried out on a new hydraulic machine, the experiment team has been presented the following data

Thrust force F $M^1 L^1 T^{-2}$

The flow velocity V $L^1 T^{-1}$

Viscosity μ $M L^{-1} T^{-1}$

Density ρ $M L^{-3}$

Size of system L L, are given

Produce meaning full dimensionless ratios

Solution:

The dimension less ratio should work equally for both model and prototype

- i. $F = f(v, \mu, \rho, L)$
- ii. $F = K (V^x \mu^y \rho^z L^w)$ K is a dimension less constant
- iii. $(M^1 L^1 T^{-2}) = [(LT^{-1})^x (M^1 L^{-1} T^{-1})^y (M L^{-3})^z (L)^w]$

Using the principles of dimensional homogeneity

$$\begin{cases} x = 2 - y, \\ z = 1 - y, \\ w = 2 - y \end{cases}$$

There fore, $F = K (V^{2-y}, \mu^y, \rho^{1-y}, L^{2-y})$

$$iv. \quad F = V^2 \rho L^2 C \left(\frac{\mu}{V \rho L} \right)^{-y} = \rho V^2 L^2 \phi \left(\frac{\rho V L}{\mu} \right)$$

$$v. \quad \frac{F}{\rho V^2 L^2} = \phi \left(\frac{\rho V L}{\mu} \right) = \Phi (Re)$$

On the basis of dimensional analysis $\frac{F}{\rho V^2 L^2}$ is dependent only on the Reynolds number.

2. BUCKINGHAM - π THEOREM

The Buckingham - pi method is widely used in the dimensional analysis of a problem and expresses the resulting equation in terms of dimensionless groups (π terms).

- ☺ It arranges variables in dimensionless groups.
- ☺ It reduces the number of variable.

Rules:

(a) If a phenomenon involves n variables and these variables are described by m fundamental dimensions, they will produce n - m groups

$$f_1 (\pi_1, \pi_2, \pi_3 \dots \pi_{n-m}) = \text{constant};$$

(b) Each π - group should be a function of maximum of m+1 variable

(c) m repeating variables are selected from amongst the n - variables so that in combination contain m fundamental dimensions

Selection of repeating variables

Repeating variables are those which will appear in all or most of the π groups, and have an influence in the problem. Before commencing the analysis of a problem, one must choose the repeating variables. There is considerable freedom allowed in the choice.

Some rules which should be followed are:

1. From the 2nd theorem there can be m (= 3) repeating variables.

2. When combined, these repeating variables must contain all of the dimensions (M, L, T)
(That is not to say that each must contain M, L and T).
3. A combination of the repeating variables must not form a dimensionless group.
4. The repeating variables do not have to appear in all π groups.
5. The repeating variables should be chosen to be measurable in an experimental investigation. They should be of major interest to the designer. For example, pipe diameter (dimension L) is more useful and measurable than roughness height (also dimension L).

In fluids it is usually possible to take ρ, μ and d as the three repeating variables.

That is describing geometry of flow, flow characteristics, and fluid property.

(d) The governing quantities must not combine among themselves to form dimensionless groups. And each π -term is dimensionless; the final function must be dimensionless, and therefore dimensionally homogeneous.

$$f(\Pi_1, \Pi_2, \Pi_3, \dots, \Pi_{n-m}) = M^0 L^0 T^0$$

From the above example F, v, μ, ρ, L

$n = 5, m = 3$ so $n-m=2$ groups (2 π -groups)

The repeating variables are ρ, L and v .

$$\pi_1 = v^{x_1} \rho^{y_1} L^{z_1} F \Rightarrow M^0 L^0 T^0 = \left(\left[\frac{L}{T} \right]^{x_1}, \left[\frac{M}{L^3} \right]^{y_1}, [L]^{z_1}, \left[\frac{ML}{T^2} \right] \right)$$

$$\pi_2 = v^{x_2} \rho^{y_2} L^{z_2} \mu \Rightarrow M^0 L^0 T^0 = \left(\left[\frac{L}{T} \right]^{x_2}, \left[\frac{M}{L^3} \right]^{y_2}, [L]^{z_2}, \left[\frac{M}{LT} \right] \right)$$

After equating both equations:

$$\pi_1 = \frac{F}{\rho v^2 L^2}$$

$$\pi_2 = \frac{\mu}{\rho v L}$$

$$\phi \left[\frac{F}{\rho v^2 L^2}, \frac{\mu}{\rho v L} \right] = 0$$

$$\text{or } \frac{F}{\rho v^2 L^2} = \phi \left[\frac{\rho v L}{\mu} \right]$$

$$\text{OR} \quad F = \rho V^2 L^2 \phi \left[\frac{\rho V L}{\mu} \right]$$

Example-

- 1) Using the variables, Q , D , $\Delta H/L$, ρ , μ , g as pertinent to smooth pipe flow, arrange them in to dimension less parameters by the Buckingham's π theorem.

Solution:

- The pertinent variables are
 Q , D , $\Delta H/L$, ρ , μ , and g
- Functional relation
 $f(Q, D, \Delta H/L, \rho, \mu, g)$
- Select the repeating variables
 D , ρ , Q
- Write the pi – parameters

2.3. SIMILITUDE AND HYDRAULIC MODELS

Hydraulic Models

The design of a major hydraulic system may be approached

- ☺ By theoretical reasoning and use of numerical models $Q = V \cdot A$.
- ☺ On the basis of previous experience of similar system.
- ☺ By scale model experiments true distorted.

It is known that even with modern computing facilities, many complex problems still challenge complete theoretical analysis. A combination of past experience, theory and dimensional analysis will provide partial or complete solution to a number of problems. However, there still remain many problems, which are tractable only through experimentation. This will be done through model studies of proposed hydraulic structures and machines.

Models permit visual observation of the flow and make it possible to obtain numerical data.

E.g. Calibrations of weirs and gates, depth of flow, velocity distributions, forces on gates, efficiencies and capacities of pumps and turbines, pressure distribution and losses.

The main objectives achieved through dimensional analysis and subsequent model testing may be:

- To obtain a relation ship amongst variables influencing a problem in terms of dimensionless parameters. The knowledge of this relation ship is essential for planning and conducting the model tests and there after in presenting the test results.

- To obtain experimental data from model tests for predicting the performance of its prototype under different conditions of flow it may be subjected to during its operation.
- To check the validity of assumptions made in the design of certain hydraulic structures. The design of many of hydraulic structure is usually based on certain assumptions, which may not be fully satisfied. It is always in the best interest, to find out whether the assumptions made are on the safer side and also to ascertain whether the structures so designed will serve the desired purpose.

The model may be larger, smaller or even of the same size as the prototype depending on type of fluid used. The choice of the fluid and the geometrical scale will depend only by the practical considerations. If complete similarity is to exist between the flow in the prototype and the flow in its model, every dimensionless parameter referring to the conditions in the models must have the same numerical value as the corresponding parameters referring to the prototype.

Hydraulic models may be either true or distorted. True models have all significant characteristic of the prototype reproduced to scale (geometrically similar) and satisfy discharge restrictions (kinematically and dynamically similar).

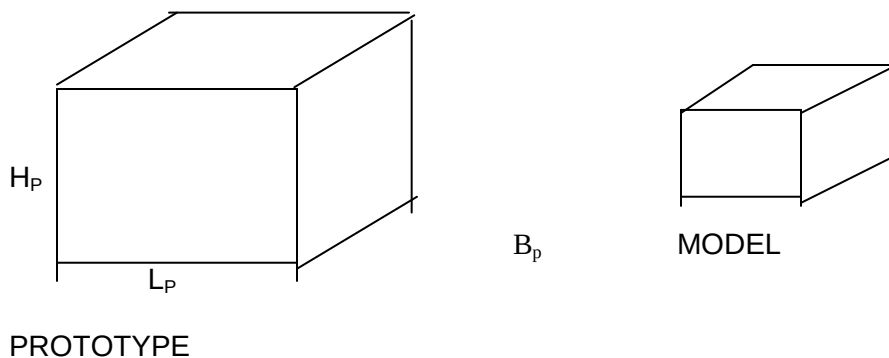
The models in which it is not possible to maintain geometric similarity is known as distorted models

TYPES OF SIMILARITY

For complete similarity to exist between the model and prototype, they must be geometrically, kinematically and dynamically similar.

GEOMETRIC SIMILARITY

It is the similarity of the shape (scale factor). It is obtained when the solid boundaries that control the follow of fluid are geometrically similar. The model is a geometric reduction of the prototype and is accomplished by maintaining a fixed ratio of all homologous lengths between the model and prototype. These physical quantities are length, area, diameter, volume, etc.



Model scale ratio:

$$L_r = \frac{L_p}{L_m} = \frac{B_p}{B_m} = \frac{H_p}{H_m} \quad (1)$$

Area Ratio:

$$A_r = \frac{A_p}{A_m} = \frac{L_p * B_p}{L_m * B_m} = L_r^2$$

Volume ratio:

$$V_r = \frac{L_p * B_p * H_p}{L_m * B_m * H_m} = L_r^3$$

KINEMATIC SIMILARITY

It is the similarity of motion. For kinematic similarity to exist, the streamline pattern in the model must be the same as in its prototype. The ratios of kinematic quantities representing the flow characteristics such as, time, velocity, acceleration, and discharge must be the same at all corresponding points.

The velocity ratio is:

$$V_r = \frac{V_p}{V_m}$$

Time scale ratio:

$$T_r = \frac{T_p}{T_m} = \frac{L_r}{V_r} \quad \{b/c \ T = L/V\}$$

Acceleration scale ratio:

$$a_r = \frac{a_p}{a_m} = \frac{L_r}{T_r^2} = \frac{V_r^2}{L_r} \quad \{b/c \ a = [L/T^2]\}$$

Discharge scale ratio:

$$Q_r = \frac{Q_p}{Q_m} = \frac{L_p^3 / T_p}{L_m^3 / T_m} = \frac{L_r^3}{T_r}$$

DYNAMIC SIMILARITY

It is the Similarity of forces involved in motion. Dynamic Similarity is attained if the ratio of homologous forces in the model and prototype are kept constant.

$$\text{i.e. } \frac{F_p}{F_m} = F_r$$

The conditions required for complete similarity are developed from the Newton 2nd law of motion $\sum F_x = ma_x$

The forces acting may be any one or a combination of several of the following: viscous, pressure, gravity, elasticity, surface tension, inertia forces etc.

$$\frac{\sum \text{Forces}(\text{viscous} + \text{pressure} + \text{gravity} + \text{elasticity} + \text{surface tension})_p}{\sum \text{Forces}(\text{viscous} + \text{pressure} + \text{gravity} + \text{elasticity} + \text{surface tension})_m} = \frac{m_p a_p}{m_m a_m}$$

$$\sum \vec{F} = \vec{F}_v + \vec{F}_g + \vec{F}_s + \vec{F}_p + \vec{F}_e = \text{RESULTANT} = -\vec{F}_i = m \cdot \vec{a}$$

NOTE: Newton's Law: Inertia force (F_i) is equal and opposite to the resultant forces.

$$\frac{(\sum F)_p}{(\sum F)_m} = \frac{(m \cdot a)_p}{(m \cdot a)_m} = \frac{(F_i)_p}{(F_i)_m} \quad (1)$$

However, in practice, a model is designed to study the effects of only a few dominant forces. Dynamic similarity requires that the ratios of these forces be kept the same between the model and prototype.

$$\text{Inertia force, } F_i = m \cdot a = \rho \ell^3 \left(\frac{v^2}{\ell} \right) = \rho \ell^2 v^2$$

$$\text{Viscous force, } F_v = \mu \frac{du}{dy} A = \mu \frac{v}{\ell} * \ell^2 = \mu \cdot v \cdot \ell$$

$$\text{Gravity force, } F_g = m \cdot g = \rho \cdot \ell^3 g$$

$$\text{Pressure force, } F_p = p \cdot A = P \cdot \ell^2$$

$$\text{Elastic force, } F_e = E_v \cdot A = E_v \cdot \ell^2$$

$$\text{Surface tension force, } F_s = \sigma \ell$$

In problems of fluid flow, the inertia force will always exist and hence it is customary to find out the force ratios with respect to the inertia forces, thus:

Inertia- to- viscous forces ratio

$$\left(\frac{F_i}{F_v} \right)_m = \left(\frac{F_i}{F_v} \right)_p \quad (2)$$

Inertia-to gravity forces ratio

$$\left(\frac{F_i}{F_g} \right)_m = \left(\frac{F_i}{F_g} \right)_p \quad (3)$$

Inertia to elastic forces ratio

$$\left(\frac{F_i}{F_e} \right)_m = \left(\frac{F_i}{F_e} \right)_p \quad (4)$$

Inertia- to surface tension forces ratio

$$\left(\frac{F_i}{F_s} \right)_m = \left(\frac{F_i}{F_s} \right)_p \quad (5)$$

Inertia -to pressure forces ratio

$$\left(\frac{F_i}{F_p} \right)_m = \left(\frac{F_i}{F_p} \right)_p \quad (6)$$

☺ When the two systems are geometrically, kinematically and dynamically similar, then they are said to be completely similar or complete similitude exists b/n the two systems.

The above six equations are dimensionless groups. The significance of the dimensionless ratios is discussed below:

(a) **REYNOLDS NUMBER** (phenomenon governed by viscous force)

A fluid in motion always involves inertia forces. If the inertial forces and viscous forces (example pipe flow) can be considered to be the only forces that govern the motion, the ratio of these forces acting on homologous particles in a model and its prototype is defined by the Reynolds number.

$$\begin{aligned} R_e &= \frac{F_i}{F_v} = (\text{Inertial force})/(\text{Viscous force}) \\ &= \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho}{\mu} L V = \frac{V L}{\nu} \quad (\text{Non dimensional ratio}) \end{aligned}$$

- This is for flow of fluid in pipe. And also for airplane traveling at speed below that at which compressibility of the air is appreciable. Further, for a submarine submerged far enough so as not to produce waves on the surfaces.

This states that when the inertial force and the viscous force are considered to be the only forces governing the motion, the Reynolds number of the model and of the prototype or of two pipelines of different fluids, must be kept constant. Thus:

$$\left(\frac{L V}{\nu} \right)_m = (R_e)_m = (R_e)_p = \left(\frac{L V}{\nu} \right)_p$$

NOTE: D is taken as L for pipe flow.

(b) **FROUDE NUMBER:** (phenomenon governed by gravity force)

When inertial forces and gravity forces are considered to be the only dominant forces in the fluid motion, the ratio of inertia forces to gravity forces acting on the homologous elements of the fluid in the model and prototype are considered as follows:

$$\left(\frac{F_i}{F_g} \right) = \frac{\rho L^2 V^2}{\rho g L^3} = \left[\frac{V^2}{gL} \right]$$

The square root of this ratio $\left(\frac{V}{\sqrt{gL}} \right)$ is known as Froude number.

Therefore, $F_r = \frac{V}{\sqrt{gL}}$

- This is used for the wave action setup by a ship, the flow of water in open channels, the forces of a stream on a bridge pier, the flow of jet from an orifice, and so on.

In open hydraulic structures

- ① Spill way
- ① Weirs
- ① Channel transitions.
- ① Sluices etc

For dynamically similitude model and prototype:

$$\left(\frac{V}{\sqrt{gL}} \right)_m = F_{r(model)} = F_{r(proto)} = \left(\frac{V}{\sqrt{gL}} \right)_p$$

N.B For open channel case L is the depth of the channel.

For a ship, L is taken as the length at water line.

- ❖ In some flow situations fluid friction is a factor of gravity as well as Inertia. Hence, Reynolds number & Froude number must both be satisfied simultaneously. To satisfy this, we have to use fluids of different viscosity for both model and prototype. Thus:

$$\frac{v_m}{v_p} = \left(\frac{L_m}{L_p} \right)^{3/2}$$

- The scale number for Froude number similarity will be:

$$V_r = \frac{V_p}{V_m} = \sqrt{L_r} \quad (\text{for the same } F_r \text{ and } g).$$

- Time ratio: $T_r = \frac{T_p}{T_m} = \frac{L_r}{V_r} = \sqrt{L_r}$

- Acceleration ratio: $a_r = \frac{V_r}{T_r} = 1$

- Discharge ratio: $Q_r = V_r A_r = L_r^{5/2}$

(c) **WEBER NUMBER** (phenomenon governed by surface tension)

The surface tension is a measure of energy level on the surface of a liquid body. The force is of primary importance in hydraulic engineering practice in the study of small surface waves or control of evaporation from a large body of water, such as water storage tank / reservoir.

In river and harbor models reduction of scale often leads to appreciable viscous and capillary effects in the shallow regions of flow. The depth of flow in such cases should be sufficiently large so that capillary effects are negligible.

The ratio of Inertia to Surface tension forces in prototype and model is:

$$\frac{F_i}{F_s} = \frac{\rho V^2 L^2}{\sigma L} = \frac{V^2}{\sigma / \rho L}$$

- ⊕ The square root of this dimensionless ratio is known as WEBER NUMBER (W_e):

$$W_e = \frac{V}{\sqrt{\sigma / \rho L}}$$

- ⊕ It is applied at the leading edge of a very thin sheet of liquid flowing over a surface. Like:

- ① Capillary movement of water in solids.
- ① Flow of liquid at a very small depth over a surface.
- ① Flow over weir at very small heads.
- ① Spray of liquid from the exit of discharging tube resulting in the formation of drops of liquids.

(d) **MACH NUMBER** (phenomenon governed by elastic forces)

The Mach number can be regarded as the ratio of inertia and elastic forces. In problems where the compressibility of the fluid becomes important, the elastic force must be considered. The high-speed motion through air causes compressibility effect (elastic force). The Mach number is therefore, measures of the effects of compressibility.

- ① Aerodynamic testing.
- ① Flow gases exceeding the velocity of sound.
- ① Water hammer problems. (Design of surge tanks).

The ratio of Inertia forces & elastic forces in prototype and model is:

$$\frac{F_i}{F_e} = \frac{\rho L^2 V^2}{E L^2} = \frac{V^2}{E / \rho}$$

- ⊕ The square root of this dimensionless number is known as MACH NUMBER (M). Thus:

$$M = \frac{V}{\sqrt{E / \rho}} = \frac{V}{C}$$

- ⊕ This is for fluid velocity (or velocity of the body through a stationary fluid) to that of a sound wave in the same medium.
- ⊕ C is the sonic velocity (or celerity) in the given medium.

(e) **EULER NUMBER** (Phenomenon governed by pressure forces.)

The ratio of Inertia forces to pressure forces for both prototype and model is given by:

$$\frac{F_i}{F_p} = \frac{\rho V^2 L^2}{PL^2} = \frac{V^2}{P/\rho}$$

✚ The square root of this dimensionless number is known as EULER NUMBER (E_u). Thus:

$$E_u = \frac{V}{\sqrt{P/\rho}} = \frac{V}{\sqrt{2\Delta P/\rho}} = \frac{V}{\sqrt{2g(\Delta P/\gamma)}}$$

Examples

- 1) A 1:10 scale model of water supply piping system is to be tested at 20°C to determine the total head loss in the prototype that carries water at 85°C. The prototype is designed to carry 5.0 m³/s discharge with 1m diameter pipes. Determine the model discharge and model velocity. Discuss how losses determined from the model are converted to proto type loss.
- 2) An over flow spillway is designed to be 100m high and 120m long, carrying a discharge of 1200 m³/s under an approaching head of 2.75m. The spillway operation is to be analyzed by a 1:50 model in a hydraulic laboratory. Determine
 - ☞ The model discharge,
 - ☞ If the discharge coefficient at the model crests measures 2.12, what is the prototype crest discharge coefficient?
 - ☞ If the velocity at the outlet of the model spill way measures 25m/s, what is the prototype velocity?
- 3) A 1:50 scale model is constructed to study a gate prototype that is designed to drain a reservoir. If the model reservoir is drained in 5.2 min, how long should it take to drain the reservoir?
- 4) A 1 m long 1:50 model is used to study the wave force on a prototype of a sea wall structure. If the total wave force measured on the model is 2.27 N and the velocity scale is 1: 10, determine the force per unit length of the prototype.

Similarity Laws or Model laws

The results obtained from the model tests may be transferred to the prototype by the use of model laws, which may be developed, from the principle of dynamic similarity.

- a) **Reynold's Model laws:** inertia & viscous forces are the only predominant forces. The similarity of flow in the model & its prototype can be established if the Reynold's number is the same for both systems.

$$(\mathbf{Re})_{\text{model}} = (\mathbf{Re})_{\text{prototype}}$$

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p} \Rightarrow \frac{\rho_p}{\rho_m} \cdot \frac{V_p}{V_m} \cdot \frac{L_p}{L_m} \cdot \frac{\mu_m}{\mu_p} = 1$$

$$\Rightarrow \frac{\rho_r \cdot V_r \cdot L_r}{\mu_r} = 1 = \frac{V_r \cdot L_r}{\nu_r}$$

- b) **Froude Model Law:** when gravitational force is added to the inertia force, the only predominant force which controls the motion, the similarity of flow in any two such systems (model & prototype) can be established if the Froude number for both the system is the same.

$$(F_r)_{\text{model}} = (F_r)_{\text{prototype}}$$

$$\frac{V_m}{\sqrt{g_m L_m}} = \frac{V_p}{\sqrt{g_p L_p}}$$

$$\frac{V_r}{\sqrt{g_r L_r}} = 1 \text{ or } V_r = \sqrt{g_r L_r} = \sqrt{L_r}$$

(For most of the case $g_r = g_p / g_m = 1$)

- c) **Euler Model Law**

$$(\mathbf{Eu})_{\text{model}} = (\mathbf{Eu})_{\text{proto}}$$

$$\frac{V_m}{\left(\frac{P_m}{\rho_m}\right)^{1/2}} = \frac{V_p}{\left(\frac{P_p}{\rho_p}\right)^{1/2}} \Rightarrow \frac{V_r}{\left(\frac{P_r}{\rho_r}\right)^{1/2}} = 1$$

$$V_r = \sqrt{\frac{P_r}{\rho_r}}$$

TYPES OF MODELS

In general hydraulic models can be classified under two broad categories

- I) Undistorted models
- II) Distorted models

I) Undistorted Models: - if a model is geometrically similar to its prototype, it is known as undistorted models. i.e., the scale ratios for the corresponding linear dimension are the same.

II) Distorted models: - if one or more terms of the models are not identical with the prototype it is known as distorted models. The distortion may be geometrical, or material or hydraulic quantities or a combination of these.

In **geometrical distortion**, the distortion can be either of dimension or that of configuration.

When different scale ratios are adopted for the longitudinal, transverse, & vertical dimensions; then it is known as **distortion of dimensions**.

It is adopted in river models where a different slope ratio for depth is adopted.

The **distortion of configuration** results when the general configuration of the model doesn't have resemblance with its prototype. If

a river model has different bed slope ratio, this is distortion of configuration.

The **material distortion** is occurred when the physical properties of the material used in the model and prototype are different.

The **distortion of hydraulic quantities** is occurred for certain uncontrollable hydraulic quantities such as time, discharge etc

Distorted models are required to be prepared for rivers, dams across very wide rivers, harbors, and estuaries etc. for which the horizontal dimensions are large in proportion to the vertical ones.

The following are some of the reasons for adopting distorted models:

- a) to maintain accuracy in vertical measurements;
- b) to maintain turbulent flow;
- c) to obtain suitable bed material & its adequate movement;
- d) to obtain suitable roughness condition;
- e) to accommodate the available facilities such as space, money, water supply & time.